Exercise 34

Find an equation of the tangent line to the curve at the given point.

$$y = x^2 \ln x, \quad (1, 0)$$

Solution

The aim is to find the slope of the tangent line at $x = 1$, so start by taking the derivative of the function with respect to x by using the product rule.

$$y' = \frac{d}{dx}(x^2 \ln x)$$

$$= \left[\frac{d}{dx}(x^2) \right] \ln x + x^2 \left[\frac{d}{dx}(\ln x) \right]$$

$$= (2x) \ln x + x^2 \left(\frac{1}{x} \right)$$

$$= 2x \ln x + x$$

$$= x(2 \ln x + 1)$$

Set $x = 1$ to get $y'(1)$.

$$y'(1) = 1(2 \ln 1 + 1) = 2(0) + 1 = 1$$

Then use the point-slope formula to get the equation of the tangent line.

$$y - 0 = y'(1)(x - 1)$$

$$y - 0 = 1(x - 1)$$

$$y = x - 1$$